Interaction-induced criticality in $\mathbb{Z}_2$ topological insulators

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Critical phenomena and quantum phase transitions are paradigmatic concepts in modern condensed matter physics. A central example in the field of mesoscopic physics is the localization-delocalization (metal-insulator) quantum phase transition driven by disorder — the Anderson transition.[1] Although the notion of localization has appeared half a century ago, this field is still full of surprising new developments. The most recent arenas where novel peculiar localization phenomena have been studied are graphene[2] and topological insulators,[3,4,5,6,7] i.e., bulk insulators with delocalized (topologically protected) states on their surface. Besides exciting physical properties, the topological protection renders such systems promising candidates for a variety of prospective electronic and spintronic devices. It is thus of crucial importance to understand properties of boundary metallic modes in the realistic systems when both disorder and interaction are present. Here we find a novel critical state which emerges in the bulk of two-dimensional quantum spin Hall (QSH) systems and on the surface of three-dimensional topological insulators with strong spin-orbit interaction due to the interplay of nontrivial $\mathbb{Z}_2$ topology and the Coulomb repulsion. At low temperatures, this state possesses a universal value of electrical conductivity. In particular, we predict that the direct QSH phase transition occurs via this novel state. Remarkably, the interaction-induced critical state emerges on the surface of a three-dimensional topological insulator without any adjustable parameters. This "self-organized quantum criticality" is a novel concept in the field of interacting disordered systems.

The universality of critical phenomena has been studied both in the area of strongly correlated systems and in mesoscopics. It is now established that disordered electronic systems can be classified into 10 symmetry classes (for review see Ref. 1). Very generally, the localization properties are determined by the symmetry class and dimensionality of the system. The critical behavior of a system depends also on the underlying topology. This is particularly relevant for topological insulators.[3,4,5,6,7,8,9,10,11,12,13]

The famous example of a topological insulator is a two-dimensional (2D) system on one of quantum Hall plateaus in the integer quantum Hall effect (QHE). Such a system is characterized by an integer (Chern number) $n = ..., -2, -1, 0, 1, 2, ...$ which counts the edge states. Here the sign determines the direction of chiral edge modes. The integer quantum Hall edge is thus a topologically protected 1D conductor realizing the group $\mathbb{Z}$.

Another ($\mathbb{Z}_2$) class of topological insulators[3,4,5] can be realized in systems with strong spin-orbit interaction and without magnetic field — and was discovered in HgTe/HgCdTe structures in Ref. 6 (see also Ref. 8). Such systems were found to possess two distinct insulating phases, both having a gap in the bulk electron spectrum but differing by the edge properties. While the normal insulating phase has no edge states, the topologically nontrivial insulator is characterized by a pair of mutually time-reversed delocalized edge states penetrating the bulk gap. Such state shows the quantum spin Hall (QSH) effect which was theoretically predicted in a model system of graphene with spin-orbit coupling.[4,14] The transition between the two topologically nonequivalent phases (ordinary and QSH insulators) is driven by inverting the band gap.[5] The $\mathbb{Z}_2$ topological order is robust with respect to disorder: since the time-reversal invariance forbids backscattering of the edge states at the boundary of QSH insulators, these states are topologically protected from localization.

A related three-dimensional (3D) $\mathbb{Z}_2$ topological insulator was discovered in Ref. 7 where crystals of $Bi_{1-x}Sb_x$ were investigated. The boundary in this case gives rise to a 2D topologically protected metal. Similarly to 2D topological insulators, the inversion of the 3D band gap induces an odd number of the surface 2D modes.[15] These states in BiSb have been studied experimentally in Refs. 7,9 and 10. Other examples of 3D topological insulators include BiTe and BiSe systems.[11,12,13]

Both in 2D and 3D, $\mathbb{Z}_2$ topological insulators are band insulators with the following properties: (i) time reversal invariance is preserved (unlike ordinary quantum Hall systems); (ii) there exists a topological invariant, which is similar to the Chern number in QHE; (iii) this invariant belongs to the group $\mathbb{Z}_2$ and reflects the presence or absence of delocalized edge modes (Kramers pairs).[3] For the sake of completeness we overview the full classification of topological insulators and superconductors[16] in Supplementary Information.
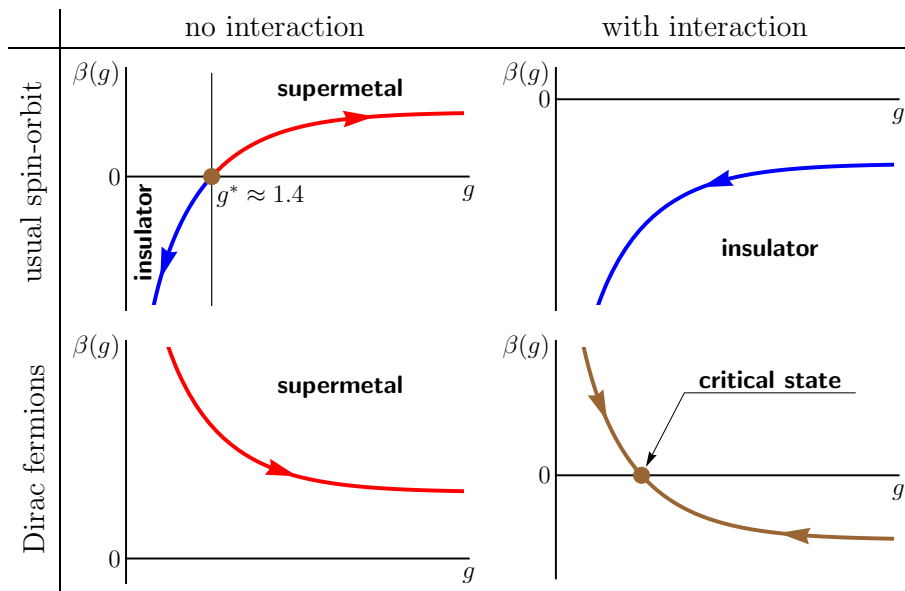
In this paper, we consider the effect of interactions

FIG. 1: Schematic scaling functions for the conductivity of 2D disordered systems of symplectic symmetry class. The plotted beta functions $\beta(g) = dg/d\ln L$ determine the flow of the dimensionless conductivity g with increasing system size L (as indicated by the arrows). The upper two panels show the beta functions for ordinary spin-orbit systems which are not topologically protected (left: no interaction; right: Coulomb interaction included). The lower two panels demonstrate the scaling for topologically protected Dirac fermions (left: no interaction; right: Coulomb interaction included).

on $\mathbb{Z}_2$ topological insulators belonging to the symplectic symmetry class, characteristic to systems with strong spin-orbit interaction. In particular, we predict a novel critical state which emerges in a 2D system due to the interplay of nontrivial topology and the Coulomb interaction.

Let us start with reviewing the localization properties of 2D systems of symplectic symmetry class AII without Coulomb interaction. In conventional spin-orbit systems (e.g. semiconductors with spin-orbit scattering), there are two phases: metal and insulator with the Anderson transition between them, see Fig. 1. A qualitatively different situation occurs in a single species of massless Dirac fermions in a random scalar potential. This system also belongs to the symplectic symmetry class but its metallic phase is "topologically protected" whatever disorder strength. This state has been recently predicted for disordered graphene with no spin- and no valley-mixing.[17,18,19] The absence of localization in this model as well as the "supermetal" scaling (Fig. 1) have been confirmed in numerical simulations.[20,21] Although a genuine single Dirac fermion cannot be realized in a truly 2D microscopic theory because of the famous "fermion doubling" problem, it emerges on the surface of a 3D topological insulator.[15]

The 3D topological insulators are characterized by the inverted sign of the gap (band inversion). This generates the surface states, as was first pointed out in Ref. 22. The effective 2D surface Hamiltonian has a Rashba form (see the derivation in Supplementary Information) and describes a single species of 2D massless Dirac particles

(cf. Ref. 23). It is thus analogous to the Hamiltonian of graphene with just a single valley. In the absence of interaction, the conductivity of the disordered surface of a 3D topological insulator therefore scales to infinity with increasing the system size. This behavior defines the topologically-protected "supermetallic phase" discussed above.

Let us now "turn on" the Coulomb interaction between electrons. Since a topological insulator is characterized by the presence of non-localizable surface states, its robustness with respect to interaction can be reformulated in terms of the absence of localization of the boundary states. At this point it is worth recalling the celebrated example of a 2D topological insulator, the QHE insulator, in which the interaction cannot destroy the chiral 1D edge modes on the boundary of a 2D QHE sample. Furthermore, the two consequent QHE topological insulators (QHE plateaus) are separated by a delocalized (critical) 2D state. Since the topological insulator phases are robust, the interaction is not capable of localizing electrons in this state separating the two topological insulators.

Having in mind this analogy with the QHE, one can expect that the delocalized states in $\mathbb{Z}_2$ topological insulators are not destroyed by the interaction either. Indeed, arguments in favor of the stability of $\mathbb{Z}_2$ topological insulators with respect to interactions were given in Refs. 3 and 24. Below we demonstrate the $\mathbb{Z}_2$ topological order and discuss its implications in 2D and 3D interacting systems in the presence of disorder.

We first consider the interacting massless Dirac electrons on the surface of a 3D topological insulator. With-
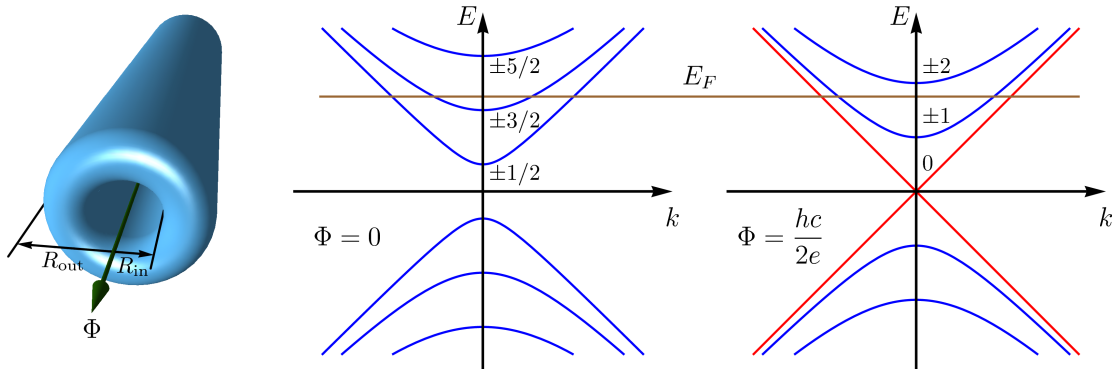
FIG. 2: Left: schematic illustration of the hollow cylinder sample used for proving the absence of localization on the surface of a 3D topological insulator (see Supplementary Information). Right: the energy spectra of a clean 2D system on the surface of the cylinder with zero flux and with half of the magnetic flux quantum penetrating the cylinder.

out interaction, the surface states are delocalized in the presence of arbitrarily strong potential disorder. In Supplementary Information we demonstrate that the interaction cannot fully localize the surface Dirac fermions. This is achieved by considering the topological insulator of a hollow cylinder geometry threaded by half of the magnetic flux quantum, see Fig. 2. Our 2D problem then reduces to the quasi-1D model with an odd number of channels. Full 2D localization would be in contradiction with known results on the absence of localization in such quasi-1D symplectic wires. Since delocalization in quasi-1D geometry survives in the presence of interaction, this is also true for the 2D interacting Dirac electrons on the surface of a 3D topological insulator.

Can the topologically protected 2D state be a supermetal ($g \to \infty$) as in the noninteracting case? To answer this question we employ the perturbative renormalization group (RG) applicable for large conductivity $g \gg 1$.

It is well known that in a 2D diffusive system the interaction leads to logarithmic corrections to the conductivity.[25] These corrections (together with the interference-induced ones) can be summed up with the use of RG technique.[26,27,28] The one-loop equation for renormalization of the conductivity in the symplectic class with Coulomb interaction has the following form:

$$\beta(g) = \frac{dg}{d\ln L} = \frac{N}{2} - 1 + (N^2 - 1)\mathcal{F}, \qquad (1)$$

where N is the number of degenerate species ("flavors": spin, isospin, ...) and L is the system size.

The first term, $N/2$, describes the effect of weak antilocalization due to disorder (this term exists also in the absence of interaction) for N parallel conductors. The second term, -1, is induced by the Coulomb interaction in the singlet channel and has a localizing effect: it suppresses the conductivity. The last term on the r.h.s. of Eq. (1) is due to the interaction in the multiplet (in the flavor space) channel. This term yields a positive (antilocalizing) correction to the conductivity. The multiplet interaction parameter $\mathcal{F}$ is itself subject to renormalization.[26]

In the degenerate case $N > 1$ (as, for example, in graphene with no spin- and valley-mixing where $N = 4$), the beta function (1) is positive corresponding to the "supermetal" phase. The situation is essentially different for 2D states on the surface of a 3D $\mathbb{Z}_2$ topological insulator where we have a symplectic system with $N = 1$. According to Eq. (1), the negative interaction-induced term in $\beta(g)$ now dominates while the multiplet term is absent. Therefore, for $g \gg 1$ the conductance decreases upon renormalization. This means that due to interaction the supermetal fixed point becomes repulsive.

Thus, on one hand, we encounter the tendency to localization due to the interaction. On the other hand, the states on the surface of the topological insulator are topologically protected from the localization. At $g \sim 1$ the topological protection reverses the sign of the β function, similarly to the ordinary QHE. As a result, a critical point emerges due to the combined effect of interaction and topology, see Fig. 1. This type of criticality should be contrasted with the QHE criticality which exists already without interactions. In our case, even in the absence of the critical state in a non-interacting model, the criticality is inevitably established in the realistic interacting systems. This novel interaction-induced critical state is the major result of our paper.

Let us now return to the case of 2D $\mathbb{Z}_2$ topological insulators. As discussed above, without interaction disorder was found to induce a metallic phase separating the two (QSH and ordinary) insulating phases.[29,30] The conductance in the metallic phase scales to infinity because of the weak antilocalization specific to the symplectic symmetry class. The schematic phase diagram for the noninteracting case is shown in Fig. 3 (left panel).

What would change in this phase diagram when interaction is taken into account? The answer follows from Eq. (1). The 2D disordered QSH system contains only a single flavor of particles, $N = 1$. Indeed, the spin-orbit interaction breaks the spin-rotational symmetry, whereas the valleys are mixed by disorder. As a result, the supermetal phase does not survive in the presence of Coulomb
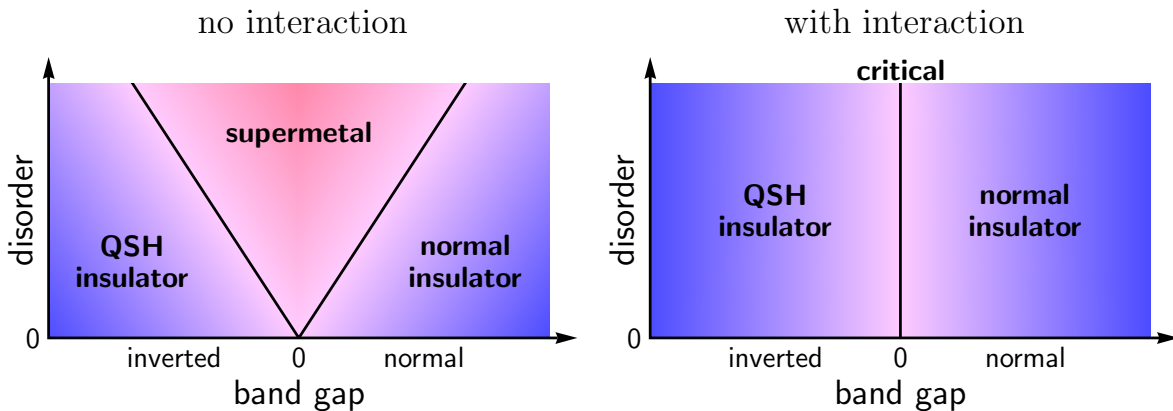
no interaction

with interaction

FIG. 3: The phase diagrams of a disordered 2D system demonstrating the QSH effect. Left: noninteracting case. In contrast to the clean case, the two topologically nonequivalent phases (QSH and ordinary insulators) have been found to be separated by the metallic phase.[29,30] The transition[1] between metal and any of the two insulators occurs at the critical value of dimensionless (in units e^2/h) conductivity $g = g^* \approx 1.4$; both transitions are believed to belong to the same universality class. For $g < g^*$ all bulk states are eventually localized in the limit of large system, while for $g > g^*$ antilocalization drives the system to the "supermetallic" state, $g \to \infty$. Right: interacting case (with Coulomb interaction not screened by external gates). Interaction "kills" the supermetallic phase. As a result, the two insulating phases are separated by the critical line.

interaction: at $g \gg 1$ the interaction-induced localization wins. This is analogous to the case of the surface of a 3D topological insulators discussed above.

The edge of a 2D topological insulator is protected from the full localization, as was discussed already in the pioneering works by Kane and Mele.[3,4] In the presence of interaction, the counter-propagating edge modes constitute the Luttinger liquid. As disorder-induced backscattering in this liquid is forbidden by the time-reversal symmetry, the dimensionless conductance of this 1D system is equal to unity. This means that the topological distinction between the two insulating phases (ordinary and QSH insulator) is not destroyed by the interaction, whereas the supermetallic phase separating them disappears. Therefore we conclude that the transition between two insulators occurs through an interaction-induced critical state, see Fig. 3 (right panel).

We have thus found that in the two types of systems with strong spin-orbit coupling the Coulomb interaction induces novel 2D critical states. This happens, first, at the boundary of 3D topological insulators and, second, in the bulk of 2D QSH systems, where the critical state separates the two topologically distinct insulating phases. In the first case the system can be described by a 2D interacting symplectic sigma-model with the $\mathbb{Z}_2$ topological term. The two critical states have much in common: (i) symplectic symmetry, (ii) $\mathbb{Z}_2$ topological protection, (iii) interaction-induced criticality, and (iv) conductivity of order unity (probably universal). This suggests that the corresponding fixed points might be equivalent. The existence of the proposed critical states can be verified in transport experiments on the semiconductor structures with possible gap inversion.

I. SUPPLEMENTARY INFORMATION

Classification of topological insulators

The full classification (periodic table) of topological insulators and superconductors for all ten symmetry classes[31,32] was developed in Refs. 34 and 33. In this Section we overview the classification of topological insulators closely following Refs. 34 and 33 and discuss the connection between the classification schemes of these papers. Very generally, the classification of topological insulators in dimension D can be constructed by studying the Anderson localization problem in a D-1 disordered system: absence of localization of surface states implies the topological character of the insulator.[33]

All symmetry classes of disordered systems[31,32] can be divided into two groups: {A,AIII} and {all other}. The classes of the big group are labeled by $p = 0, 1, \ldots, 7$. Each class is characterized by (i) Hamiltonian symmetry class H_p; (ii) symmetry class of the classifying space used by Kitaev[34] R_p; (iii) symmetry class S_p of the compact sector $\mathcal{M}_F$ of the sigma-model manifold. The symmetry class R_p of the classifying space of reduced Hamiltonians characterizes the space of matrices obtained from the Hamiltonian by keeping all eigenvectors and replacing all positive eigenvalues by $+1$ and all negative by -1. Note

TABLE I: Symmetry classes and "Periodic Table" of topological insulators.[33,34] The first column enumerates the symmetry classes of disordered systems which are defined as the symmetry classes H_p of the Hamiltonians (second column). The third column lists the symmetry classes of the classifying spaces (spaces of reduced Hamiltonians).[34] The fourth column represents the symmetry classes of a compact sector of the sigma-model manifold. The fifth column displays the zeroth homotopy group $\pi_0(R_p)$ of the classifying space. The last four columns show the possibility of existence of $\mathbb{Z}$ and $\mathbb{Z}_2$ topological insulators in each symmetry class in dimensions $d = 1, 2, 3, 4$.

	Symmetry classes				Topological insulators			
p	H_p	R_p	S_p	$\pi_0(R_p)$	d=1	d=2	d=3	d=4
0	AI	BDI	CII	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$
1	BDI	BD	AII	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0
2	BD	DIII	DIII	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0
3	DIII	AII	BD	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0
4	AII	CII	BDI	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$
5	CII	C	AI	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$
6	C	CI	CI	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$
7	CI	AI	C	0	0	0	$\mathbb{Z}$	0
$0'$	A	AIII	AIII	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$
$1'$	AIII	A	A	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0

that

$$R_p = H_{p+1}, \quad S_p = R_{4-p}. \tag{2}$$

Here and below cyclic definition of indices $\{0, 1, \ldots, 7\}$ (mod 8) and $\{0', 1'\}$ (mod 2) is assumed.

For the classification of topological insulators it is important to know homotopy groups π_d for all symmetry classes. In Table I we list $\pi_0(R_p)$; other π_d are given by

$$\pi_d(R_p) = \pi_0(R_{p+d}). \tag{3}$$

The homotopy groups π_d have periodicity 8 (Bott periodicity).

There are two ways to detect topological insulators: by inspecting the topology of (i) classifying space R_p or of (ii) the sigma-model space S_p.

(i) Existence of topological insulator (TI) of class p in d dimensions is established by the homotopy group π_0 for the classifying space R_{p-d}:

$$\begin{cases} \text{TI of the type } \mathbb{Z} \\ \text{TI of the type } \mathbb{Z}_2 \end{cases} \iff \pi_0(R_{p-d}) = \begin{cases} \mathbb{Z} \\ \mathbb{Z}_2 \end{cases} \tag{4}$$

(ii) Alternatively, the existence of topological insulator of symmetry class p in d dimensions can be inferred from the homotopy groups of the sigma-model manifolds, as follows:

$$\begin{cases} \text{TI of the type } \mathbb{Z} \iff \pi_d(S_p) = \mathbb{Z} \\ \text{TI of the type } \mathbb{Z}_2 \iff \pi_{d-1}(S_p) = \mathbb{Z}_2 \end{cases} \tag{5}$$

This criterium is obtained if one requires existence of "non-localizable" boundary excitations. This may be guaranteed by either Wess-Zumino term in $d - 1$ dimensions [which is equivalent to the $\mathbb{Z}$ topological term in d dimensions, i.e. $\pi_d(S_p) = \mathbb{Z}$] for a QHE-type topological insulator, or by the $\mathbb{Z}_2$ topological term in $d - 1$ dimensions [i.e. $\pi_{d-1}(S_p) = \mathbb{Z}_2$] for a QSH-type topological insulator.

The above criteria (i) and (ii) are equivalent, since

$$\pi_d(S_p) = \pi_d(R_{4-p}) = \pi_0(R_{4-p+d}). \tag{6}$$

and

$$\pi_0(R_p) = \begin{cases} \mathbb{Z} & \text{for } p = 0, 4, \\ \mathbb{Z}_2 & \text{for } p = 1, 2. \end{cases} \tag{7}$$

In this paper we focus on 2D systems of symplectic (AII) symmetry class. One sees that this is the only symmetry class out of ten classes that supports the existence of $\mathbb{Z}_2$ topological insulators both in 2D and 3D. The effect of interaction on $\mathbb{Z}_2$ topological insulators and superconductors in classes DIII (in 2D) and CII (in 3D) will be considered elsewhere.

Surface states of 3D topological insulators

In this section we consider the surface of 3D topological insulators and the derive the effective surface Hamiltonian. The realistic microscopic Hamiltonian of such systems with strong spin-orbit interaction can be modelled by the 3D massive Dirac Hamiltonian, see, e.g., Refs. 15 and 16. We start with the general form of a 3D Dirac Hamiltonian

$$H_{3D} = \begin{pmatrix} -M & \sigma\mathbf{p} \\ \sigma\mathbf{p} & M \end{pmatrix}. \tag{8}$$

This Hamiltonian is a matrix in the 4×4 space formed by the spin and pseudospin (sublattice) spaces. The interface between the semiconductor and the vacuum is described by sending the mass M to infinity.

Let us consider a flat interface at $x = 0$. The edge state with zero energy decays into the bulk ($x < 0$):

$$\Psi = e^{Mx} \begin{pmatrix} \psi \\ \chi \end{pmatrix}. \tag{9}$$

Acting by the Hamiltonian (8) on this wavefunction, we obtain the relation between the two components of the spinor, $\chi = i\sigma_x \psi$. For an arbitrary surface characterized by the normal vector $\mathbf{n}$, the general boundary condition reads

$$\chi = i\sigma \mathbf{n} \psi. \tag{10}$$

From Eqs. (8) and (10) we obtain the effective 2D surface Hamiltonian in the form of the Rashba Hamiltonian in a curved space

$$H_{\text{surf}} = \frac{i}{2} [\sigma \mathbf{p}, \sigma \mathbf{n}] = \frac{\nabla \mathbf{n}}{2} + \frac{1}{2} (\mathbf{n}[\mathbf{p} \times \sigma] + [\mathbf{p} \times \sigma]\mathbf{n}). \tag{11}$$

This 2D Hamiltonian describes a single species of 2D massless Dirac particles and is thus analogous to the Hamiltonian of graphene with just a single valley.

Absence of localizations of 2D surface states

Consider the 2D system formed at the surface of a 3D topological insulator and described by the Hamiltonian (11). Let us first prove the absence of localization in the non-interacting case. Assume that all the states in 2D are localized with some localization length ξ. Consider a hollow cylinder with all dimensions much larger than ξ pierced by the Aharonov-Bohm magnetic flux $\Phi = hc/2e$ (half of the flux quantum). This value of Φ does not break the time-reversal symmetry leaving the system in the symplectic class. In the absence of disorder we can characterize the surface states by the momentum k along the cylinder axis and by the integer angular momentum n. The energy of such a state is given by

$$E = \sqrt{k^2 + (n/R)^2}. \tag{12}$$

The channels with positive and negative n are degenerate, while $n = 0$ channel is not. Thus the cylinder sustains an odd number of conducting channels both on the inner and outer surface at any value of chemical potential.

Let us now include disorder and show the absence of localization in quasi-one-dimensional (q1D) symplectic system with odd number of channels. The scattering matrix of such a q1D wire has the form

$$S = \begin{pmatrix} r & t' \\ t & r' \end{pmatrix} \tag{13}$$

with transmission and reflection amplitudes as its entries. The blocks r and r' are square matrices of the size determined by the number of channels. Time-reversal symmetry of the symplectic type imposes the following restrictions on the amplitudes entering the matrix S:

$$r = -r^T, \quad r' = -r'^T, \quad t = t'^T. \tag{14}$$

Calculating the determinant of the both sides of the first identity and taking into account the odd size of the matrix r, we obtain $\det r = 0$. This implies a zero eigenvalue of r and hence the existence of a channel with perfect transmission. We conclude: in a q1D wire of symplectic symmetry with an odd number of channels one channel always remains delocalized.[35,36,37,38]

Applying the q1D result to the cylinder constructed above we immediately come to the controversy: in spite of assumed 2D localization on the surface, the infinitely long cylinder possesses two (inner and outer) conducting channels. This proves the absence of localization in 2D.

The proof can be generalized to include the Coulomb interaction. We assume the temperature to be much smaller than the inverse time of electron propagation through the system. At such low temperatures the inelastic scattering of electrons is negligible and we can describe the transport by the scattering matrix calculated at the Fermi energy and accounting for virtual processes. The symmetry properties of this S matrix are unchanged and hence the above proof applies.

[1] F. Evers and A. D. Mirlin, Rev. Mod. Phys. **80**, 1355 (2008).

[2] A. H. Castro Neto, F. Guinea, N. M. R. Peres, K. S. Novoselov, and A. K. Geim, Rev. Mod. Phys. **81**, 109 (2009).

[3] C. L. Kane and E. J. Mele, Phys. Rev. Lett. **95**, 146802 (2005).

[4] C. L. Kane and E. J. Mele, Phys. Rev. Lett. **95**, 226801 (2005).

[5] B. A. Bernevig, T. L. Hughes, and S.-C. Zhang, Science **314**, 1757 (2006).

[6] M. König, S. Wiedmann, C. Brüne, A. Roth, H. Buhmann, L. W. Molenkamp, X.-L. Qi, and S.-C. Zhang, Science **318**, 766 (2007).

[7] D. Hsieh, D. Qian, L. Wray, Y. Xia, Y. S. Hor, R. J. Cava, and M. Z. Hasan, Nature **452**, 970 (2008).

[8] A. Roth, C. Brüne, H. Buhmann, L. W. Molenkamp, J. Maciejko, X.-L. Qi, and S.-C. Zhang, Science **325**, 294 (2009).

[9] D. Hsieh, Y. Xia, L. Wray, D. Qian, A. Pal , J. H. Dil, J. Osterwalder, F. Meier, G. Bihlmayer, C. L. Kane, Y. S. Hor, R. J. Cava, and M. Z. Hasan, Science **323**, 919 (2008).

[10] P. Roushan, J. Seo, C. V. Parker, Y. S. Hor, D. Hsieh, D. Qian, A. Richardella, M. Z. Hasan, R. J. Cava, and A. Yazdani, Nature **460**, 1106 (2009).

[11] D. Hsieh, Y. Xia, D. Qian, L. Wray, J. H. Dil, F. Meier, J. Osterwalder, L. Patthey, J. G. Checkelsky, N. P. Ong, A. V. Fedorov, H. Lin, A. Bansil, D. Grauer, Y. S. Hor, R. J. Cava, and M. Z. Hasan, Nature **460**, 1101 (2009).

[12] H. Zhang, C.-X. Liu, X.-L. Qi, X. Dai, Z. Fang, and S.-C. Zhang, Nature Phys. **5**, 438 (2009)

[13] Y. L. Chen, J. G. Analytis, J.-H. Chu, Z. K. Liu, S. K. Mo, X. L. Qi, H. J. Zhang, D. H. Lu, X. Dai, Z. Fang, S. C. Zhang, I. R. Fisher, Z. Hussain, and Z.-X. Shen, Science **325**, 178 (2009).

[14] L. Sheng, D. N. Sheng, C. S. Ting, and F. D. M. Haldane, Phys. Rev. Lett. **95**, 136602 (2005).

[15] L. Fu and C. L. Kane, Phys. Rev. B **76**, 045302 (2007).

[16] A. P. Schnyder, S. Ryu, A. Furusaki, and A. W. W. Ludwig, Phys. Rev. B **78**, 195125 (2008).

[17] P. M. Ostrovsky, I. V. Gornyi, and A. D. Mirlin, Phys. Rev. Lett. **98**, 256801 (2007).

[18] P. M. Ostrovsky, I. V. Gornyi, and A. D. Mirlin, Eur. Phys. J. Special Topics **148**, 63 (2007).

[19] S. Ryu, C. Mudry, H. Obuse, and A. Furusaki, Phys. Rev. Lett. **99**, 116601 (2007).

[20] J. H. Bardarson, J. Tworzydlo, P. W. Brouwer, and C. W. J. Beenakker, Phys. Rev. Lett. **99**, 106801 (2007).

[21] K. Nomura, M. Koshino, and S. Ryu, Phys. Rev. Lett. **99**, 146806 (2007).

[22] M. I. Dyakonov and A. V. Khaetskii, JETP Lett. **33**, 115 (1981).

[23] B. A. Volkov and O. A. Pankratov, JETP Lett. **42**, 178 (1985).

[24] S.-S. Lee and S. Ryu, Phys. Rev. Lett. **100**, 186807 (2008).

[25] B.L. Altshuler and A.G. Aronov, in *Electron-electron interactions in disordered conductors*, edited by A.L. Efros and M. Pollak (Elsevier, 1985), p. 1.

[26] A. M. Finkelstein, Sov. Sci. Rev. A. Phys. **14**, 1 (1990).

[27] A. Punnoose and A. M. Finkel'stein, Phys. Rev. Lett. **88**, 016802 (2001).

[28] D. Belitz and T. R. Kirkpatrick, Rev. Mod. Phys. **66**, 261 (1994).

[29] M. Onoda, Y. Avishai, and N. Nagaosa, Phys. Rev. Lett. **98**, 076802 (2007).

[30] H. Obuse, A. Furusaki, S. Ryu, and C. Mudry, Phys. Rev. B **76**, 075301 (2007).

[31] M. R. Zirnbauer, J. Math. Phys. **37**, 4986 (1996).

[32] A. Altland and M. R. Zirnbauer, Phys. Rev. B **55**, 1142 (1997).

[33] A. P. Schnyder, S. Ryu, A. Furusaki, and A. W. W. Ludwig, AIP Conf. Proc. **1134**, 10 (2009).

[34] A. Yu. Kitaev, AIP Conf. Proc. **1134**, 22 (2009).

[35] M.R. Zirnbauer, Phys. Rev. Lett. **69**, 1584-1587 (1992).

[36] A.D. Mirlin, A. Müller-Groeling, and M.R. Zirnbauer, Ann. Phys. **236**, 325 (1994).

[37] T. Ando and H. Suzuura, J. Phys. Soc. Jpn. **71**, 2753 (2002).

[38] Y. Takane, J. Phys. Soc. Jpn. **73**, 1430 (2004).